

Considering Late-Time Acceleration in some Cosmological Models

S. Davood Sadatian

Abstract We study two cosmological models: A non-minimally coupled scalar field on brane world model and a minimally coupled scalar field on Lorentz invariance violation model. We compare some cosmological results in these scenarios. Also, we consider some types of Rip singularity solution in both models.

Keywords Scaler-Vector-Tensor Theories, Theories of Extra Dimensions, Brane world Cosmology, Late-time Acceleration, DGP Brane Cosmology, Lorentz Invariance Violation.

1 Introduction

In theories of extra spatial dimensions, ordinary matter is captured on the brane but gravitation extends through the entire space-time (Arkani-Hamed et al. 1998; Randall & Sundrum 1999; Dvali et al. 2000). In these scenarios, the cosmological evolution on the brane is taken by an effective Friedmann equation that combines the effects of the bulk in a non-trivial kind (Binetruy et al. 2000a,b; Maartens 2004). In other view point, the important result in brane models is an alternative scenario for late-time expansion of the universe. This result predicts deviations from the 4-dimensional gravity at limited distances. In other hand, the model considered by Dvali, Gabadadze and Porrati (DGP) is different from above models since it also predicts deviations from the standard 4-dimensional gravity in large distances (Binetruy et al. 2000a; Collins & Holdom 2000). Generally one can study the effect of a caused gravity term as a quantum

discipline in any brane world model. The existence of a higher dimensional embedding space lets for the bulk or brane matter that can evidently affect the cosmological evolution on the brane (Langlois & Rodriguez-Martinez 2001). A special form of bulk or brane context is a scalar field. Scalar fields take an important key both in models of the early universe and late-time acceleration. These scalar fields give a dynamical model for matter fields in a brane world scenarios (Bogdanos et al. 2006; Farakos & Pasipoularides 2006; Mizuno et al. 2003; Bouhamdi-Lopez & Wands 2005).

In other hand, Lorentz invariance violation models (LIV) has been considered in the scalar-vector-tensor theories (Kanno & Soda 2006). It has described that Lorentz violating vector fields influence the dynamics equations in the inflationary models. An interesting result of this model is that the exact Lorentz violating inflationary solutions are depended on the absence of the inflation potential. In this case, the inflation is exactly collaborated with the Lorentz violation (Arianto et al. 2007). Therefore, we study this symmetry breaking on the dynamics of equation of state for some cosmological aspects.

Some evidences from supernova data (Perlmutter et al. 1999; Riess et al. 1998), (CMB) results (Miller et al. 1999; de Bernardis et al. 2000; Hanany et al. 2000) and (WMAP) data (Spergel et al. 2003; Page et al. 2003; Spergel et al. 2007; Reiss et al. 2004; Zhao et al. 2012), point out an accelerating phase of cosmological expansion and this characterize shows that the picture of universe by pressureless fluid is not enough; the universe should contain some type of additional negative pressure known as dark energy (For brief introduction in this field see (Arianto et al. 2007)). Also, the merged analysis of the WMAP data with the supernova Legacy survey (SNLS) (Spergel et al. 2007), compels the equation of state w_{de} , in accord with 74% donation of dark energy in the currently accelerating universe. More-

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over, observations show some sort of a dark energy equation of state, $w_{de} < -1$ (Reiss et al. 2004). Hence, a practical cosmological model should accept a dynamical equation of state that may have crossed the value $w_{de} = -1$, in late time of cosmological evolution.

In following, we consider cosmological results of a non-minimally coupled scalar field on the brane and a minimally coupled scalar field in LIV model. Also we determine late-time behavior of our equations and obtain some restriction on the parameters of models to have an accelerating universe.

We understand that dark energy is an important problem in modern cosmology, especially, if the equation of state parameter ω less than -1 . In this article, we study basically solving Friedman equations and obtain the evolution of the effective equation of state parameter ω . The phantom crossing conditions that we obtained for these set-ups, themselves are interesting at some degree. Furthermore, we know that just considering background quantity is not enough in such kind of unconventional cosmological models. The current constraint on ω is obtained by combining the result of the observation of CMB, which means that the information of the evolution of perturbations should be also included. Also we understand in the brane world model, the perturbations of brane are coupled with these of bulk, which gives nontrivial effect. Therefore, in first stage we just study the background dynamics and the evolution of perturbations will study in future. In other view, there is a no-go theorem which explicitly points out a conventional dark energy model involving single degree of freedom in the frame of standard Einstein gravity is forbidden to realize such a scenario. This no-go theorem was proven in the appendix of (Xia et al. 2008). In this regards, we point out the original work in (Feng et al. 2005) and several papers addressing this scenario (Cai et al. 2007; Cai & Wang 2008; Cai et al. 2010). This theorem is exactly the reason why we study a number of nonconventional dark energy models in realizing the Quintom scenario (Zhao et al. 2012), such as the non-minimally coupled on brane world model and the Lorentz-violating model considered in the present paper. However, a *non-minimally* coupling scalar field identified on the brane model and scalar field coupling *minimally* to gravity in LIV model have some similar cosmological results, this is an interesting theoretical motivation of these models. Also we study other solutions admitted as Rip singularity, that occur in the condition $\omega < -1$ increases rapidly. However, it possible different types of singularity, depending energy density and scale factor how increases with time (Frampton et al. 2011, 2012).

2 Non-minimally coupled scalar Field on the Brane

Here we study a brane world model where a scalar field is coupling non-minimally to the Ricci scalar of the brane. In following we only consider a scalar field in the matter Lagrangian without taking into account baryons, cold dark matter, and radiation. This kind of analysis suffers from a potential maybe cause that the model not be able to explain the evolution of a realistic universe at background level when confronting with observations. However, this problem can solve by suitable fine-tuning parameters of model.

The action in the absence of ordinary matter can be given as (Bouhamdi-Lopez & Wands 2005; Myung 2001)

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{k_4^2} \alpha(\phi) R[g] - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right], \quad (1)$$

where we have made a general non-minimal coupling $\alpha(\phi)$. For simplicity, in following we set $k_4^2 \equiv 8\pi G_N = 1$. We can obtain Einstein equations with variation of the action respect to brane metric

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \alpha^{-1} \mathcal{T}_{\mu\nu} \quad (2)$$

where $\mathcal{T}_{\mu\nu}$, energy-momentum tensor of the scalar field non-minimally coupled to gravity is taken by

$$\begin{aligned} \mathcal{T}_{\mu\nu} = & \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 - g_{\mu\nu} V(\phi) \\ & + g_{\mu\nu} \square \alpha(\phi) - \nabla_\mu \nabla_\nu \alpha(\phi), \end{aligned} \quad (3)$$

where \square shows 4-dimensional d'Alembertian. We study the FRW universe with line element as following

$$ds^2 = -dt^2 + a^2(t) d\Sigma_k^2. \quad (4)$$

where $d\Sigma_k^2$ is the line element for a constant curvature $k = +1, 0, -1$. The Ricci scalar obtain with the equation of motion for scalar field as

$$\nabla^\mu \nabla_\mu \phi = V' - \alpha' R[g], \quad (5)$$

where a prime denotes the derivative of each parameter with respect to ϕ , that may be written by

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} + \frac{dV}{d\phi} = \alpha' R[g]. \quad (6)$$

where a dot means the derivative of each parameter by respect to t The intrinsic Ricci scalar for a FRW brane

give as

$$R[g] = 6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right), \quad (7)$$

and Friedmann's equations are determined by

$$\frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{\rho}{3}, \quad (8)$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p). \quad (9)$$

We take a scalar field, ϕ , only depended on time. So, with Eq(3), we obtain

$$\rho = \alpha^{-1}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\alpha'H\dot{\phi}\right), \quad (10)$$

$$p = \alpha^{-1}\left(\frac{1}{2}\dot{\phi}^2 - V(\phi) + 2(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2)\right) \quad (11)$$

where $H = \frac{\dot{a}}{a}$ is Hubble parameter. Now equation of state has the following form

$$w \equiv \frac{p}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi) + 4(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha'H\dot{\phi}}. \quad (12)$$

When $\dot{\phi} = 0$, we have $p = -\rho$. In this illustration ρ depended on a and $V(\phi)$, it has the duty of a cosmological constant. In the minimal case which $\dot{\phi}^2 < V(\phi)$, by Eq(9), we take $p < -\frac{\rho}{3}$ that shows a late-time accelerating universe. In non-minimal case, the position depends on the choice of non-minimal coupling parameter. In following we show how for a suitable range of coupling parameter, a late-time accelerating expansion can be described. We first consider a moving domain wall of brane world to discuss quintessence behavior, then we study a special non-minimal coupling for late-time acceleration.

2.1 Late-Time Acceleration in a Brane world Model

We study a bulk config by two 5-dimensional anti de Sitter-Schwarzschild (AdS₅) black hole spaces combined on a *moving domain wall*. To insert this moving domain wall into 5-dimensional bulk, it is required to indicate normal and tangent to the domain wall by determination of normal instructing to the brane. We take that domain wall is identified at coordinate $r = a(\tau)$ where $a(\tau)$ is considered by Israel junction

conditions (Carroll 2004). Here we study the following line element(Myung 2001)

$$ds_{5\pm}^2 = -\left(k - \frac{\eta_{\pm}}{r^2} + \frac{r^2}{\ell^2}\right)dt^2 + \frac{1}{k - \frac{\eta_{\pm}}{r^2} + \frac{r^2}{\ell^2}}dr^2 + r^2\gamma_{ij}dx^i dx^j, \quad (13)$$

where \pm is for left(-) and right(+) side of the moving domain wall, also ℓ is curvature radius of AdS₅ manifold and γ_{ij} is the horizon metric. $\eta_{\pm} \neq 0$ creates the electric part of the Weyl tensor on two sides(Myung 2001).

We assume usual mater on the brane has a perfect fluid form, $T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + ph_{\mu\nu}$ where $\rho = \rho_m + \sigma$ and $p = p_m - \sigma$. Energy density of the confined matter on the brane and analogous pressure are shown with ρ_m and p_m , while σ is the brane tension. with Israel junction conditions(Carroll 2004) and Gauss-Codazzi equations, we obtain generalized Friedmann equations(Myung 2001) as following

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{\rho_m}{3} + \frac{\eta}{a^4} + \frac{\ell^2}{36}\rho_m^2 \quad (14)$$

$$\frac{\ddot{a}}{a} = -\frac{\rho_m}{6}(1 + 3w) - \frac{\eta}{a^4} - \frac{\ell^2}{36}\rho_m^2(2 + 3w) \quad (15)$$

where we require a Z_2 -symmetry by $\eta_+ = \eta_- \equiv \eta$ and we have supposed $p_m = w\rho_m$. following we take case: $\eta = 0$. For $\eta = 0$, each sub-manifolds of bulk space-time are accurate AdS₅ space-times. Now we study a confined non-minimally coupled scalar field on the brane and consider cosmological aspects. We use energy density and pressure of scalar field (10) and (11) and by equation (15), hence for cosmic expansion have

$$\begin{aligned} \frac{\ddot{a}}{a} = & -\frac{1}{6\alpha}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\alpha'H\dot{\phi}\right) \times \\ & \left(1 + 3\frac{\dot{\phi}^2 - 2V(\phi) + 4(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha'H\dot{\phi}}\right) \\ & - \frac{\ell^2}{36\alpha^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\alpha'H\dot{\phi}\right)^2 \times \\ & \left(2 + 3\frac{\dot{\phi}^2 - 2V(\phi) + 4(\alpha'\ddot{\phi} + 2H\alpha'\dot{\phi} + \alpha''\dot{\phi}^2)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha'H\dot{\phi}}\right). \quad (16) \end{aligned}$$

where $H = \frac{\dot{a}}{a}$ is Hubble parameter. This is a complex equation, to describe cosmological aspects, we must

study some limiting cases or specify $\alpha(\phi)$, $V(\phi)$ and ϕ .

Equation (16) shows, the condition for an accelerating universe depended on the choice of non-minimal coupling and the scalar field potential. Hence we determine the following non-minimal coupling

$$\alpha(\phi) = \frac{1}{2}(1 - \xi\phi^2). \quad (17)$$

For an accelerating universe we should have $\rho_m + 3p_m < 0$ (Bouhamdi-Lopez & Wands 2005; Faraoni 2000). If we take $V > 0$, we obtain

$$(1 - 3\xi)\dot{\phi}^2 - V(\phi) - 3\xi\phi(\ddot{\phi} + H\dot{\phi}) < 0 \quad (18)$$

dynamics of this scalar field is given as

$$\ddot{\phi} + 3H\dot{\phi} + \xi R\phi + \frac{dV}{d\phi} = 0, \quad (19)$$

we obtain

$$(1 - 3\xi)\dot{\phi}^2 - V(\phi) + 3\xi^2 R\phi^2 + 6\xi H\phi\dot{\phi} + 3\xi\phi\frac{dV}{d\phi} < 0. \quad (20)$$

With ρ_m in this case, we determine

$$y \equiv (1 - \xi\phi^2)\rho_m - 2V(\phi)$$

$$+\dot{\phi}^2\left(\frac{1}{2} - 3\xi\right) + 3\xi^2 R\phi^2 + 3\xi\phi\frac{dV}{d\phi} < 0 \quad (21)$$

that is the requirement to have an accelerated universe. In following, we study the weak energy condition $\rho_m \geq 0$ and limit our consideration to the case by $\xi \leq 1/6$. Therefore we obtain

$$-2V + 3\xi\phi\frac{dV}{d\phi} < y < 0 \quad (22)$$

and a main condition for cosmic acceleration is

$$V - \frac{3\xi}{2}\phi\frac{dV}{d\phi} > 0, \quad \xi \leq \frac{1}{6}. \quad (23)$$

For the model have cosmic acceleration by $\xi > 0$, the potential $V(\phi)$ must change with ϕ slower than power-law potential $V_c(\phi) = V_0\left(\frac{\phi}{\phi_0}\right)^{\frac{2}{3\xi}}$ (Fig 1). In other hand, when $\xi < 0$, the main condition for cosmic acceleration needs V grow faster than V_c as ϕ increases (Faraoni 2000). Now we take a simple special example to show how this model process. By potential of the form $V(\phi) = \lambda\phi^n$, condition (23) takes $\lambda\left(1 - \frac{3n\xi}{2}\right) > 0$. In this case just for $\xi \leq 2/3n$ we have the accelerated expansion.

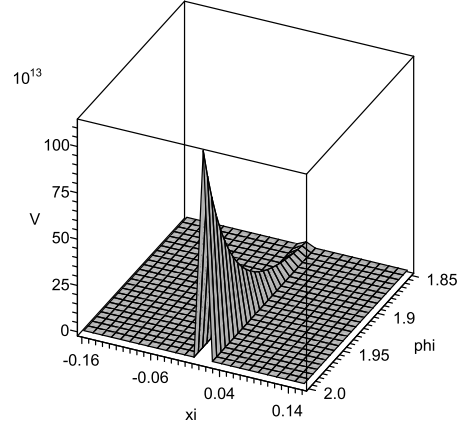


Fig. 1 V vary for different values of the ϕ and Non-minimal coupling parameter ξ for $V = \phi^{\frac{2}{3\xi}}$.

More general case is $\eta \neq 0$ in equations (14) and (15). In following we convert the generalized Friedmann equation as

$$\begin{aligned} \dot{\chi} + 4\frac{\dot{a}}{a}\left[\chi + \frac{1}{12\alpha}\left(\frac{1}{2}(1 - 2\xi)\dot{\phi}^2 - V(\phi) - 2\xi\phi\left(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi}\right)\right)\right] \\ = 0, \end{aligned} \quad (24)$$

where $\chi = \left(\frac{\dot{a}}{a}\right)^2$ is dark energy field. Then we take $\phi(t) \approx \frac{A}{t^\beta}$ where A is a constant and β an unspecified power will be studied. For scale factor we take the ansatz $a(t) \approx Bt^\nu$ where B is a constant and ν to be determined. For scalar field potential we take $V(\phi) = \lambda\frac{\phi^2}{2}$. Therefore we obtain

$$\begin{aligned} \frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 \approx \frac{\sigma^2}{12\alpha u} + \frac{\sigma}{24\alpha u}\rho(1 - 3w) + \frac{1}{6\alpha}(V(\phi) + \Lambda) \\ - \frac{\dot{\phi}^2}{12\alpha}(1 + 4\alpha'') - \frac{\alpha'}{3\alpha}\left(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi}\right) \end{aligned} \quad (25)$$

and

$$\begin{aligned} \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{u'}{2u}\dot{\phi}^2 \approx -\frac{6\alpha}{u}V' + 10\frac{\alpha'}{u}(V(\phi) + \Lambda) \\ + 4\frac{\alpha'(u + \alpha'u')\sigma}{u^3}\rho(1 - 3w) + \frac{8\alpha'(u + \alpha'u')\sigma^2}{u^3} \end{aligned} \quad (26)$$

where $u = 6\left[1 - \xi\left(1 - \frac{16}{3}\xi\right)\frac{\phi^2}{2}\right]$. We take usual matter on the brane has an equation of state such as $\rho =$

$Dt^{-3\nu(1+w)}$. Hence we have

$$\begin{aligned} \frac{\nu(2\nu-1)}{t^2} &\approx \frac{1}{24} \left\{ \frac{\sigma^2}{3} + 4\Lambda + \left(\frac{\sigma(1-3w)D}{6} \right) \frac{1}{t^{3\nu(1+w)}} + \right. \\ &\frac{A^2}{t^{2\beta}} \left[2\lambda + 2\xi \left(\Lambda + \frac{4(\frac{3}{8}-\xi)}{9} \sigma^2 \right) + \frac{4\xi D(1-3w)(\frac{3}{8}-\xi)\sigma}{9} \times \right. \\ &\left. \left. \frac{1}{t^{3\nu(1+w)}} \frac{2\beta}{t^2} \left(\beta(1-8\xi) + 4\xi(3\nu-1) \right) \right] \right\} \end{aligned} \quad (27)$$

and scalar field equation given as

$$\begin{aligned} \beta(\beta+1-3\nu) \frac{A}{t^{\beta+2}} &\approx -\frac{\lambda A}{t^\beta} \left[1 - \frac{8}{3} \xi^2 \frac{A^2}{t^{2\beta}} \right] \\ &- \frac{5\xi}{3} \frac{A}{t^\beta} \left[\Lambda + \left(\frac{\lambda}{2} + \frac{\xi}{2} \left(1 - \frac{16}{3} \xi \right) \Lambda \right) \frac{A^2}{t^{2\beta}} \right] \\ &- \frac{\xi\sigma}{9} \left(2\sigma + \frac{D}{t^2} (1-3w) \right) \frac{A}{t^\beta} \left[1 + \xi \left(1 - \frac{16}{3} \xi \right) (1+\xi) \frac{A^2}{t^{2\beta}} \right] \end{aligned} \quad (28)$$

where $\beta > 1$. Two conditions must solve numerically to limit the values of ν and β for an accelerating expansion. After some calculations we obtain

$$-\frac{(1-3w)\xi\sigma}{9} D = A\beta(\beta+1-3\nu), \quad (29)$$

$$\lambda = \frac{-\xi\sigma^2}{12}, \quad (30)$$

where we passed over terms of order $\mathcal{O}(t^{-3\beta})$ and higher. Now, if we arrange λ , since $\nu(2\nu-1) = \frac{\sigma(1-3w)}{144} D$, we find

$$\beta = \frac{3\nu-1}{2} \pm \left[\frac{(3\nu-1)^2}{2} - 16\xi\nu(2\nu-1) \right]^{\frac{1}{2}}, \quad (31)$$

where $\nu = \frac{2}{3(1+w)}$. For $\nu < 1$ the parameter β is real for any value of ξ , for $\nu > 1$ the exponent β is real only if we have

$$\xi \leq \frac{1}{16} \frac{(3\nu-1)^2}{2\nu(2\nu-1)}. \quad (32)$$

These equations limited the values of non-minimal coupling ξ for obtain an accelerating expansion (Fig 2). Hence, it seems that non-minimally coupled scalar field identified on the brane world supplies natural candidate for late-time expansion.

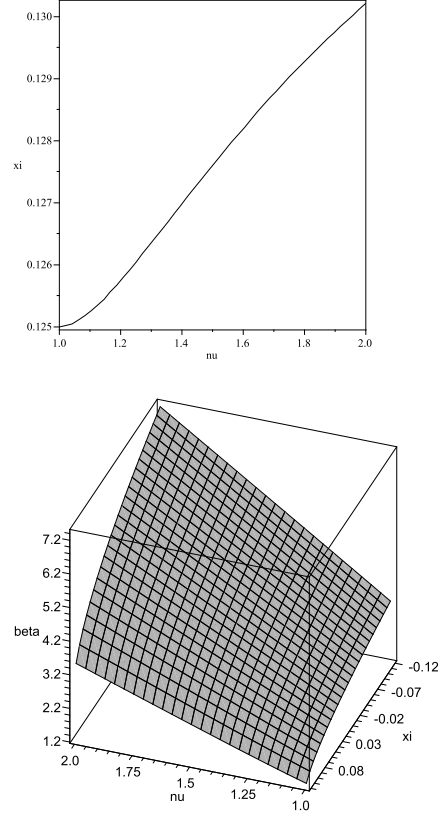


Fig. 2 Variation of ξ respect to ν with equation (32) (up) and Variation of β respect to ν and ξ with equation (31) by sign +, for sing - in this equation (31) β has negative value. We choose special range of parameter ξ with equation (32) in (down) figure.

Also we can use equation (12) for determine dynamical Equation of state $\omega(t)$.

Figure (3) shows possible crossing of phantom divided barrier in this framework, this result is important because previous considerations have shown, crossing phantom divided line with a scalar field non-minimally coupled gravity exist (Nesseris & Perivolaropoulos 2007; Vikman 2005).

In following, we explain the physical reasons why the crossing of the phantom divide can occur in this model. Here, we have a scalar field that non minimally coupled in gravity and an ordinary matter. Generally, according to (Nesseris & Perivolaropoulos 2007), these sources of energy-momentum can switch together and describe evolution of universe. In this regard, an encouraging candidate for the dark matter is either a positive cosmological constant or a slowly developing scalar known as quintessence. Also the quintessence can be determined as an alternative method to solve the cosmological constant problem. This is possible because instead of the fine-tuning, the quintessence gives a model of slowly decaying cosmological constant. However, there exists

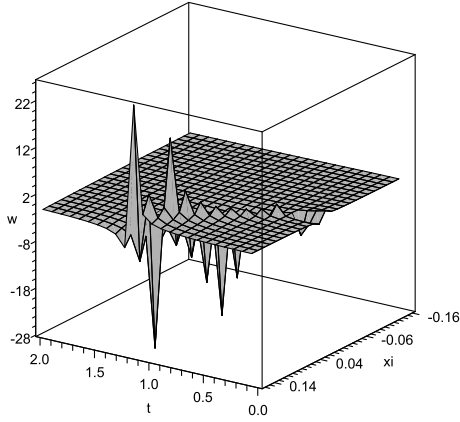


Fig. 3 Dynamical Equation of state $\omega(t)$ vary for different values of the t and Non-minimal coupling parameter ξ by equation (12) and specific choose for parameters space.

another question for the quintessence now. This is to inquire whether the expansion will keep accelerating forever or it will slow down again after some time. This is very similar to the leaving problem in the inflationary cosmology. According to the theory of quintessence, the dark energy of the universe is controlled by the potential of a scalar field ϕ which is still curling to its minimum at $V = 0$.

2.2 Rip Singularity

The new interest to the models with the crossing phantom divide $\omega < -1$ is their prediction of a Rip singularity (Starobinsky 2000). Generally, the scale factor of the universe gets infinite at a finite time in the future which was named Big Rip singularity. There were proposed some scenarios to improved the Big Rip singularity: (I) To determined phantom acceleration as temporary phenomenon. This is a number of scalar potentials. (II) To explain for quantum effects which maybe delay/stop the singularity happening (Elizalde et al. 2004). (III) To change the gravitation itself in such a way that it seems to be observationally-friendly from a side but it restores to singularity. (IV) To couple dark energy with dark matter in the specific way or to use specific form for dark energy equation of state (Bamba et al. 2008). Pay attention to for quintessence dark energy, other finite-time singularities may appear. For instance, type II (sudden) singularity or type III singularity appears with finite scale factor but infinite energy and/or pressure. The near examination shows that the situation $\omega < -1$ is not enough

for a singularity event (Nojiri et al. 2005). First of all, a transient phantom cosmology is possible. However, one can make such models that ω asymptotically tends to -1 and the energy density increases with time or corpses constant but there is no finite-time singularity (Nojiri et al. 2005). Of course, most apparent case is when Hubble rate inclined to constant (cosmological constant or asymptotically de Sitter space), which can also point out the pseudo-rip (Frampton et al. 2012). most interesting situation is connected with Little Rip cosmology (Frampton et al. 2011) where Hubble rate go to infinity in the infinite future. The goal point is that if ω approaches -1 adequately rapidly, then it is possible to have a model in which the time needed for singularity is infinite. However, this situation does Not occur in our model, because H is finite.

In following, if we want to determind some types of Rip singularity, we must use Hubble parameter, energy density and pressure equations that obtained in above equations. Therefore, after calculations with $\xi = 0.14$ (figure 4), we take results as following:

case 1 : type II singularity (sudden)

conditions: scale factor and energy density arrives finite value but $p \rightarrow \infty$

In this model, we find that scale factor, pressure and energy density are finite. Therefore, the model does not predict a sudden singularity.

case 2 : type III singularity

Conditions, scale factor arrives finite value but ($\rho \rightarrow \infty$) and $|p| \rightarrow \infty$

This means the energy density becomes so rapidly with time and scale factor does not arrive the infinite value. The goal difference between case (1) and (2) for this model is that the potential of scalar field has a stake in a case of singularity. Here, according numerical calculations, if scale factor arrives finite value, energy density and pressure are finite. Hence, type III singularity does not occur in this setup.

case 3 : pseudo-rip singularity

conditions : Pressure $p \rightarrow -\rho$ in infinite time and cause to $\dot{H} \rightarrow 0$

This means the expansion of the universe asymptotically appeals to the exponential area. Obviously in our model with above setup, pseudo-rip singularity predict, because in infinite time, when \dot{H} arrive to zero, $p \rightarrow -\rho$.

3 A Lorentz Violating Model

With using (Kanno & Soda 2006; Arianto et al. 2007; Sadatian 2012), we consider the cosmological aspects

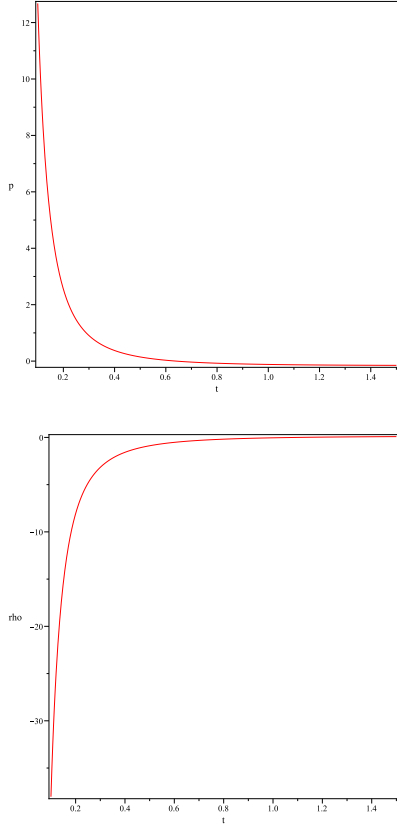


Fig. 4 Variation of pressure(up) and energy density(down) respect to time for a non minimal coupling model(where $\xi = 0.14$)

of Lorentz invariance violating model. We emphasize this model parameters are tightly bounded by both astronomical and cosmological experiments, some works such as (Arianto et al. 2007, 2008; Zen et al. 2009; Arianto et al. 2010) explained, the model parameters are able to be bounded by comparing the model with cosmological observations. Also a global analysis and a group of canonical values for the model parameters to constrain these parameters given in (Kostelecky & Samuel 1986), so that the model is expected to be viable and background evolution is reasonable.

We want to obtain a connection between Lorentz Invariance violation parameter and dynamics of scalar field. First we define an action for a representative scalar-vector-tensor theory which allows Lorentz invariance violation as

$$S = S_g + S_u + S_\phi, \quad (33)$$

where the actions for the vector field S_u , the tensor field S_g , and the scalar field S_ϕ are assume as

$$S_g = \int d^4x \sqrt{-g} \frac{1}{16\pi G} R \quad (34)$$

$$S_u = \int d^4x \sqrt{-g} [-\beta_1 \nabla^\mu u^\nu \nabla_\mu u_\nu - \beta_2 \nabla^\mu u^\nu \nabla_\nu u_\mu$$

$$- \beta_3 (\nabla_\mu u^\mu)^2 - \beta_4 u^\mu u^\nu \nabla_\mu u^\alpha \nabla_\nu u_\alpha + \lambda (u^\mu u_\mu + 1)] \quad (35)$$

$$S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi. \quad (36)$$

This action has a non-gravitational degrees of freedom in the structure of Lorentz violating scalar-tensor-vector theory. We take $u^\mu u_\mu = -1$ and the expected value of vector field u^μ is $\langle 0 | u^\mu u_\mu | 0 \rangle = -1$ (Kostelecky & Samuel 1986). $\beta_i(\phi)$ ($i = 1, 2, 3, 4$) are unrestricted parameters by dimension of mass squared, also \mathcal{L}_ϕ is the Lagrangian density for the scalar field in this model (Kanno & Soda 2006). If universe be homogeneous and isotropic, we consider the universe by metric as

$$ds^2 = -\mathcal{N}^2(t) dt^2 + e^{2\alpha(t)} \delta_{ij} dx^i dx^j, \quad (37)$$

where \mathcal{N} is a lapse function and the universe scale is given by α (Kanno & Soda 2006; Arianto et al. 2007). If the action vary with respect to metric and selecting a fitting gauge, field equations obtain as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \quad (38)$$

where $T_{\mu\nu} = T_{\mu\nu}^{(u)} + T_{\mu\nu}^{(\phi)}$ is the total energy-momentum tensor, and energy-momentum tensors of vector and scalar fields given by $T_{\mu\nu}^{(\phi)}$. The time and space elements of the total energy-momentum tensor obtain as (Arianto et al. 2007)

$$T_0^0 = -\rho_u - \rho_\phi, \quad T_i^i = p_u + p_\phi, \quad (39)$$

where the energy density and pressure of the vector field determine as

$$\rho_u = -3\beta H^2, \quad (40)$$

$$p_u = \left(3 + 2\frac{H'}{H} + 2\frac{\beta'}{\beta} \right) \beta H^2, \quad (41)$$

$$\beta \equiv \beta_1 + 3\beta_2 + \beta_3, \quad (42)$$

a prime point to the derivative of any parameters by respect to α and $H \equiv d\alpha/dt = \dot{\alpha}$ is the Hubble parameter. The energy equations for scalar field, ϕ and the vector field u given as

$$\rho'_u + 3(\rho_u + p_u) = +3H^2 \beta', \quad (43)$$

$$\rho'_\phi + 3(\rho_\phi + p_\phi) = -3H^2 \beta', \quad (44)$$

So, the total energy equation for two the vector and the scalar fields write as

$$\rho' + 3(\rho + p) = 0, \quad (\rho = \rho_u + \rho_\phi). \quad (45)$$

Now, dynamics of the model determine with the following Friedmann equations (Kanno & Soda 2006; Arianto et al. 2007)

$$\left(1 + \frac{1}{8\pi G\bar{\beta}}\right) H^2 = \frac{1}{3\bar{\beta}} \rho_\phi \quad (46)$$

$$(1 + \frac{1}{8\pi G\bar{\beta}})(HH' + H^2) = -\frac{1}{6}\left(\frac{\rho_\phi}{\bar{\beta}} + \frac{3p_\phi}{\bar{\beta}}\right) - H^2 \frac{\beta'}{\bar{\beta}}. \quad (47)$$

For the scalar section of this model we assume the following Lagrangian

$$\mathcal{L}_\phi = -\frac{\eta}{2}(\nabla\phi)^2 - V(\phi), \quad (48)$$

where $(\nabla\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Usual scalar fields are match to $\eta = 1$ while $\eta = -1$ describe phantom fields. The homogeneous scalar field has the density ρ_ϕ and pressure p_ϕ as

$$\rho_\phi = \frac{\eta}{2}H^2\phi'^2 + V(\phi), \quad (49)$$

$$p_\phi = \frac{\eta}{2}H^2\phi'^2 - V(\phi) \quad (50)$$

and equation of state parameter obtain as

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = -\frac{1 - \eta H^2\phi'^2/2V}{1 + \eta H^2\phi'^2/2V}. \quad (51)$$

Now the Friedmann equation take as (Arianto et al. 2007)

$$H^2 = \frac{1}{3\bar{\beta}} \left[\frac{\eta}{2} H^2 \phi'^2 + V(\phi) \right], \quad (52)$$

where $\bar{\beta} = \beta + \frac{1}{8\pi G}$. With this equation we obtain

$$\phi' = -2\eta\bar{\beta} \left(\frac{H_{,\phi}}{H} + \frac{\bar{\beta}_{,\phi}}{\bar{\beta}} \right). \quad (53)$$

If this equation Substituting into the Friedmann equation, we can find the potential of the scalar field as (Arianto et al. 2007)

$$V = 3\bar{\beta}H^2 \left[1 - \frac{2}{3}\eta\bar{\beta} \left(\frac{\bar{\beta}_{,\phi}}{\bar{\beta}} + \frac{H_{,\phi}}{H} \right)^2 \right]. \quad (54)$$

where $H = H(\phi(t))$. The equation of state find in following form

$$\begin{aligned} \omega_\phi &= -1 + \frac{4}{3}\eta\bar{\beta} \left(\frac{H_{,\phi}}{H} + \frac{\bar{\beta}_{,\phi}}{\bar{\beta}} \right)^2 \\ &= -1 + \frac{1}{3}\eta \frac{\phi'^2}{\bar{\beta}}. \end{aligned} \quad (55)$$

Relations (53) and (55) have main role in following considerations. We can see violation of the Lorentz invariance which has been described by existence of a vector field in the action, now it has combined in the dynamics of scalar field and equation of state by $\bar{\beta}$ parameter. This situation allows to consider phantom divided line crossing in LIV model. Now we should solve equations, (53) and (55), to obtain dynamics of scalar field ϕ and the equation of state ω_ϕ . Therefore, we must define the Hubble parameter $H(\phi)$ and the vector field, $\bar{\beta}(\phi)$. In following, we choose a general case of the Hubble parameter $H(\phi)$ and the vector field $\bar{\beta}(\phi)$ and then consider crossing of phantom divided line, late time acceleration and Rip singularity scenario.

3.1 Considering Late-Time Acceleration in LIV model

For an acceleration universe, should be $\ddot{a} > 0$. We rewrite it to this form $H'/H > -1$. For to consider this case, we must determine some equations. We study a general case for the vector field and the Hubble parameter, note that these equations are a function of ϕ (Sadatian & Nozari 2008)

$$H = H_0\phi^\zeta, \quad \bar{\beta}(\phi) = m\phi^n, \quad n > 2 \quad (56)$$

Following, we just study a quintessence scalar field by $\eta = 1$. By using relation (53), we calculate

$$\phi(t) = \frac{1}{A} \quad (57)$$

where

$$A = \left[H_0(t - t_0)(-4\zeta m + 4\zeta mn + 2\zeta^2 m - 4mn + 2mn^2) + \phi_0 \right]^{\left(\frac{1}{n+\zeta-2}\right)}$$

and by using relation (55), we can obtain

$$\omega_\phi(t) = -1 + \frac{4}{3}m\phi^{n-2}(t)(\zeta + n)^2. \quad (58)$$

Following with relations (56) and (55,58), we find

$$m^2 < \frac{1}{4(-1)^n\phi^{n-2}(t)(\zeta + n)^2}, \quad n > 2, \quad (59)$$

This equation describe a restriction in LIV parameters for a late time acceleration.

Now with using equation (56) and (57) for obtain scale factor as

$$a(t) = a_0(t_0)e^{\frac{2\phi_0^{\frac{\zeta}{n+\zeta-2}} \left(m(n+\zeta)(n+\zeta-2)H_0(t-t_0) + \frac{1}{2}\phi_0 \right) e^{F-\phi_0}}{2\phi_0^{\frac{\zeta}{n+\zeta-2}} (n-2)m(n+\zeta)}}$$

(60)

where

$$F = \zeta \ln \left(\frac{1}{e^{\frac{\ln(2m(n+\zeta)(n+\zeta-2)H_0(t-t_0)+\phi_0)}{n+\zeta-2}}} \right).$$

We should select a special space parameter, therefore we approach equation (60) by a Taylor series in special space parameter

$$a(t) = .286504 + .447663t + .237821t^2 + .0352651t^3 \quad (61)$$

We can see in figure 5 acceleration evolution in late

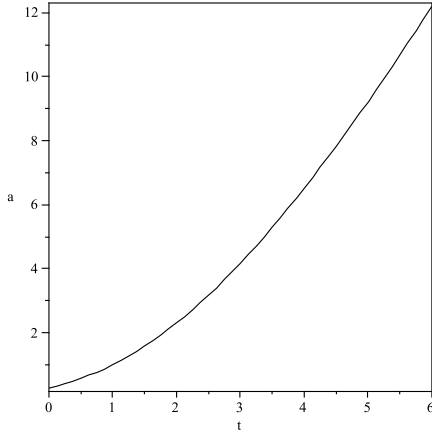


Fig. 5 Variation of scale factor $a(t)$ for different values of t for $n = 3$, $\zeta = -2$ and $m = -0.1$. The values of ζ are determined by relation (59).

time. Now, we study equation of state for determine crossing phantom divided line. By ϕ according to equation (57), the equation of state obtain as

$$\omega_\phi(t) = -1 + \frac{4}{3}m \frac{(\zeta + n)^2}{E} \quad (62)$$

where

$$E = [H_0(t - t_0)(-4\zeta m + 4\zeta mn + 2\zeta^2 m$$

$$- 4mn + 2mn^2) + \phi_0]^{\left(\frac{n-2}{n+\zeta-2}\right)},$$

which implicitly has a dynamical behavior. This model lets us to choose a suitable parameter space to crossing phantom divided line. Moreover this parameter space must be compared by observational data. We insist in this model, a scalar field and a vector field together can explain crossing phantom divided barrier and late time acceleration. We can see in figure 6, the

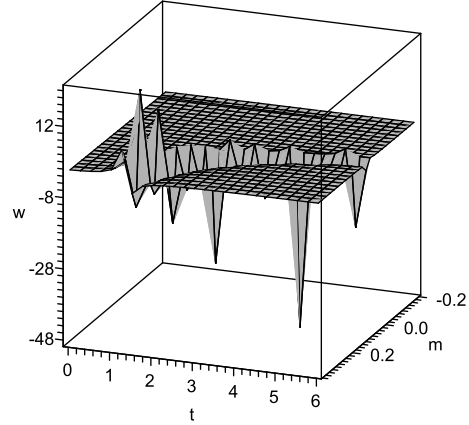


Fig. 6 ω_ϕ vary for different values of the vector field by m and t for $n = 3$ and $\zeta = -2$. Positive values of ζ does not show a phantom divided line crossing. The values of ζ are obtained with equation (59).

crossing of phantom divided line for a dynamical equation of state. Figure 6 perhaps explain why we are living in an era of $\omega < -1$. Note that $\bar{\beta}(t)$ has the main role of Lorentz invariance violation model. The dynamical equation for $\bar{\beta}(t)$ has an interested mean: with a suitable fine tuning we can obtain a Lorentz violating cosmology correspond to observational data.

We know many different models that explain phantom divided line crossing, but this model is special because it contains only a scalar field and a Lorentz violating vector field that checks the crossing (Li et al. 2008; Libanov et al. 2007; Bertolami et al. 2004). However, two notice must be explained in this paper: First, in figures 3 and 6 we can see, there are some sudden jerks of the equation of state. In some models that equation of state crossing the phantom divided line, ω surge around -1 (see (Wei 2006) and references therein). These jerks are certainly a signature of chaotic manner of equation of state. Second, in those figures we see, crossing of the phantom divided line appear at late-time. This means, as second cosmological coincidence problem, requires extra fine-tuning in model parameters.

For describing the physical reasons why the crossing of the phantom divide can occur in this model, we can state by a fitting coupling between a quintessence scalar field and other matter content may causes to a constant ratio of the energy densities of both contexts which is reconcilable with an accelerated expansion of the universe or crossing of phantom dividing line. In this model, we have three kinds of energy-momentum: 1- standard matter, 2- Scalar Field as a nominee of Dark Energy and 3- energy-momentum content rely on for Lorentz violating vector field. Here we

take standard matter has tiny contribution on the all energy-momentum content of the universe. For other two energy-momentum contents, it is possible to take the trigger mechanism to determined dynamical equation of state. This means that we take scalar- vector-tensor theory containing Lorentz invariance violation which performs like the hybrid inflation models. In this condition, vector and scalar field play the roles of inflaton and the waterfall field. Hence, it is reasonable to hope that one of them will eventually control to describe inflation or accelerating phase and crossing of phantom divided line.

3.2 Rip Singularity

According to Rip singularity scenario that explained in previous section(page 5), we consider some types of Rip singularity in this LIV model. As shown in figures (5,6), we provide suitable conditions for considering Rip singularity scenario. Hence, correspond to equation of state, and using equations such as energy density, pressure and Hubble parameter, we can study some Rip singularity solutions.

According our setup in this paper that selected $\zeta = -2$ and $n = 3$ in anzats (56) we have

$$H = H_0 \phi^{-2}, \quad \beta = m \phi^3$$

Now with substituting above equations in energy density and pressure relations, we can consider dynamical behaviors in these relations. Note that, we selected for ordinary matter $\rho_m = 0$ and $p_m = 0$. Now after numerical calculations (figure 7), we obtain results as following:

In finite time, scalar factor and energy density increases, but pressure decreases. Therefore type II singularity (sudden) do not predict. But type III singularity appears because $|p|$ decrease so rapidly. In infinite time, Hubble rate tends to zero, therefore Little Rip singularity is impossible, but pseudo-rip singularity can occur. We emphasize this relations determine in original setup in this paper. This means by different selection in parameters of ζ and m , all dynamic of relations be change. Therefore, one can change setup and consider other options. However, in this situation, our work be limited.

4 Comparison with Observational Data & Other Dark energy Models

4.1 Analyzing SNe Data

The parameters of the cosmological models may be consider from a explicit comparison of their predictions

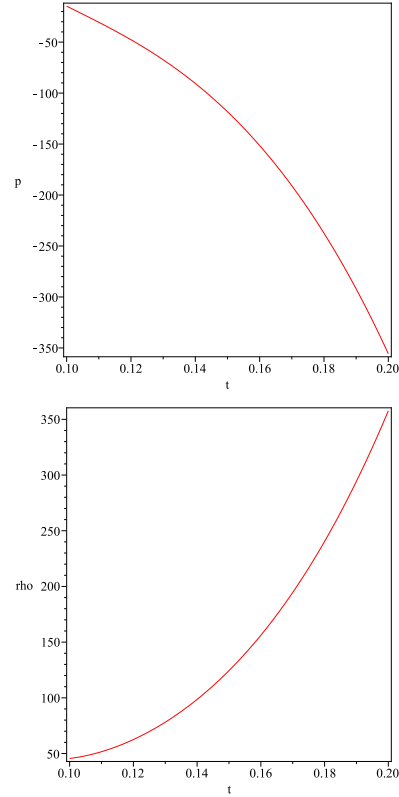


Fig. 7 Variation of pressure(up) and energy density(down) respect to time for a minimal coupling model(where $\zeta = -2$ and $n = 3$)

with precise observational data. Here we determined the data coming by SNe observations, the evolution of the Hubble parameter in the brane model.

The modulus μ vs redshift $z = a_0/a - 1$ equation corresponding to type Ia supernovae from the Supernova Cosmology Project (Amanullah et al. 2010; Uni 2013) is,

$$\mu(z) = \mu_0 + 5 \lg D_L(z).$$

The equation for the luminosity distance $D_L(z)$ as a function of the redshift in the FRW cosmology (FC) give as

$$D_L^{FC} = \frac{c}{H_0} (1+z) \int_0^z h^{-1}(z) dz, \quad (63)$$

where $h(z) = [\Omega_{m0}(1+z)^3 + \Omega_{D0}F(z)]^{1/2}$.

Here, Ω_{m0} is the total fraction of matter density, Ω_{D0} the fraction of DE energy density, and H_0 the current Hubble parameter. The constant value μ_0 conditional upon the chosen Hubble parameter:

$$\mu_0 = 42.384 - 5 \log h, \quad h = H_0/100 \text{ km/s/Mpc}$$

The function $F(z) = \rho_D(z)/\rho_{D0}$ may be consider from the continuity equation

$$\dot{\rho}_D - 3\frac{\dot{a}}{a}g(\rho_D) = 0, \quad (64)$$

which can be reobtain as

$$\int_{\rho_{D0}}^{\rho_D(z)} \frac{dy}{g(y)} = -3 \ln(1+z). \quad (65)$$

For simplicity, we ignore the contribution of radiation. For cosmology on the brane (BC), Eq.(63) can be take as

$$D_L^{BC} = \frac{c}{H_0}(1+z) \int_0^z h^{-1}(z)[1+\delta h^2(z)]^{-1/2}(1+\delta)^{1/2} dz$$

where for comfort the parameter $\delta = \rho_0/2$ has been introduced. For the analysis of the SNe data one needs to obtain the parameter χ^2 , which is given by

$$\chi_{SN}^2 = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i)]^2}{\sigma_i^2}, \quad (66)$$

where σ_i is the corresponding 1σ error. The parameter μ_0 is free of the data points and, therefore, one has to do a uniform marginalization over μ_0 . Minimization with respect to μ_0 may be done by simply enlarging the χ_{SN}^2 with respect to μ_0 ,

$$\chi_{SN}^2 = A - 2\mu_0 B + \mu_0^2 C, \quad (67)$$

where

$$A = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0)]^2}{\sigma_i^2},$$

$$B = \sum_i \frac{\mu_{obs}(z_i) - \mu_{th}(z_i)}{\sigma_i^2}, \quad C = \sum_i \frac{1}{\sigma_i^2}.$$

The expression has a minimum for $\mu_0 = B/C$ at

$$\bar{\chi}_{SN}^2 = A - B^2/C.$$

One may minimize $\bar{\chi}_{SN}^2$ instead of χ_{SN}^2 . with (Nesseris & Perivolaropoulos 2005) and table 1, one obtains the 56.01% confidence level by $\Delta\chi^2 = \chi^2 - \chi_{min}^2 < 1.01$ for the one-parameter or 1.9 for the two-parameter models. correspondingly, the 91.2% confidence level is determined by $\Delta\chi^2 = \chi^2 - \chi_{min}^2 < 3.78$ or 5.63 for the one- and two-parameter models.

4.2 Dark energy Cosmology

In this paper, we consider two models that explain dark energy in content of the universe, but according (Bamba et al. 2012) and references therein, there are many different models in this subject. Therefore, In this section for a worthy review, we briefly point out them.

As we know some number of well-liked dark energy models, such as the Λ CDM model, Pseudo-Rip Little Rip and Little Rip scenarios, the phantom and quintessence cosmology with the four types of the singularities and non-singular universes satisfied with dark energy have been considered.

Investigations have been shown the Λ CDM model and various cosmological observations to endure the bounds on the late-time acceleration of the universe. In this regards, researchers have considered a fluid depiction of the universe in which the dark fluid has a popular form of the Equation of state with the inhomogeneous. They have shown that all the dark energy cosmology can be describe by some different fluids, also determined their properties. It has also been display that at the present stage the cosmological evolutions of all the dark energy universes can be like to that of the Λ CDM scenario, and hence those models are satisfied with the cosmological observations. In other hand, they have studied the equality of appreciate of different dark energy models, this means, single and multiple scalar field theories could be portray.

However, we emphasize the role of cosmography in this review. It is a essential tool because it lets, in doctrine, to separate among models without a seizures but laying on restriction coming from data. However, the goal criticism to this approach is depended to the expansion of the Hubble parameter. In mostly, observations could not be enlarged at any red-shift and, in some of occasions, are not enough to track models up to early eras. Finally, the future observational data campaigns must fix the condition elimination the degeneration incipient at low red-shifts and let a well insight of models.

z	$H_{obs}(z)$ km s ⁻¹ Mpc ⁻¹	σ_H km s ⁻¹ Mpc ⁻¹
0.170	83	8
1.530	140	14
1.750	202	40

Table 1 Hubble parameter in contrast to redshift data.

5 Summary

Several evidences from observational data have shown that our universe is in the accelerated expansion era. In this paper, we have shown this reality in two different models: A non-minimally coupling scalar field identified on the brane and a scalar field coupling minimally to gravity in LIV model. According complex of dynamical equations, we have limited our research to some special form of non-minimal coupling and scalar field potential. Then we have studied special form of time evolution for the scale factor and a scalar field. In brane world model, we have determined a equation that explain situations where an accelerating expansion of the universe is possible. As a consequence, we have obtained which non-minimally coupling scalar field identified on the brane is a suitable candidate for late-time expansion of universe. In LIV model, we have considered a new framework for crossing of phantom divided line with equation of state, that combined a violation of Lorentz invariance in a cosmological model. However, we have explained with a suitable choice of parameter space, it is possible to have phantom divided line crossing just with a Lorentz invariance violating vector field and a single scalar field (Nesseris & Perivolaropoulos 2007; Vikman 2005). In this regard, existence of a Lorentz invariance violating vector field prepare a setup for crossing phantom divided line just with a minimally coupling scalar field. In other view point, there is the possibility of a Rip singularity by suitable tuning in the parameters of models.

Comparison of non-minimal model with WMAP data takes more precise restricts on the values of non-minimal coupling. A detailed comparison between this results and some results of previous considerations shows that our constraints for non-minimal coupling with exponential potential are steadfast by holographic dark energy and also with result of warped DGP brane model (Nesseris & Perivolaropoulos 2007).

Appendix 1: Proof of stability

Now we consider the stability of our model. In order to consider the classical stability of our model, we determined the manner of the model in the $\omega - \omega'$ plane where ω' is the derivative of ω with the logarithm of the scale factor (see (Arianto et al. 2007) for a comparable analysis)

$$\omega' \equiv \frac{d\omega}{d \ln a} = \frac{d\omega}{dt} \frac{dt}{d \ln a} = \frac{\dot{\omega}}{H}. \quad (68)$$

The sound speed declares the phase velocity of the inhomogeneous perturbations of the field. Hence, we take the function c_a as

$$c_a^2 \equiv \frac{\dot{p}}{\dot{\rho}}. \quad (69)$$

If the matter is determined as a perfect fluid, this equation could be the adiabatic sound speed of this fluid. We emphasize with scalar fields that do not comply perfect fluid form necessarily, this parameter is not actually a sound speed. The conservation of the total (scalar and vector) field given with (45). As we view, that relation implicitly depended on β and H form. Therefore, we can take anzats (56) with substituting in (45) as

$$\frac{d\rho_{total}}{dt} + 3(\rho_{total} + p_{total}) = 0 \quad (70)$$

Since the dust matter complies the continuity equation and the Bianchi identity keeps legally, total energy density fulfill the continuity equation. From that equation, we have

$$\dot{\rho}_{de} = -3\rho_{de}(1 + \omega_{de}) \quad (71)$$

where de means dark energy. By using equation of state $p_{de} = \omega_{de}\rho_{de}$, we have

$$\dot{p}_{de} = \dot{\omega}_{de}\rho_{de} + \omega_{de}\dot{\rho}_{de} \quad (72)$$

Hence, the function c_a^2 can write as

$$c_a^2 = \frac{\dot{\omega}_{de}}{-3(1 + \omega_{de})} + \omega_{de} \quad (73)$$

In this condition, with suitable choice of ζ and n in (56), which we take in page(10), we calculated the $\omega - \omega'$ plane is divided into four regions defined as

$$I : \quad \omega_{de} > -1, \quad \omega' > 3\omega(1 + \omega) \Rightarrow c_a^2 > 0$$

$$II : \quad \omega_{de} > -1, \quad \omega' < 3\omega(1 + \omega) \Rightarrow c_a^2 < 0$$

$$III : \quad \omega_{de} < -1, \quad \omega' > 3\omega(1 + \omega) \Rightarrow c_a^2 < 0$$

$$IV : \quad \omega_{de} < -1, \quad \omega' < 3\omega(1+\omega) \Rightarrow c_a^2 > 0 \quad (74)$$

As one can see from these equation, the regions I and IV have the classical stability in this model.

Appendix 2: *initial conditions for numerical calculations in both models*

A : In the brane world model

In the induced gravity model we should add the extra condition

$$\rho \gg |6\alpha(H^2 + \frac{K}{a^2})|$$

Hence we need both high energy and weak coupling. In other hand, the last condition and the continuity equation hints that $P = -\rho$ for $H \neq 0$. However that unlike 4D general relativity, we do not necessarily claim $\rho = \text{constant}$. In other view point, initial values for parameters ξ and β given from equation

$$\xi \leq \frac{1}{16} \frac{(3\nu - 1)^2}{2\nu(2\nu - 1)}$$

and

$$\beta = \frac{3\nu - 1}{2} \pm \left[\frac{(3\nu - 1)^2}{2} - 16\xi\nu(2\nu - 1) \right]^{\frac{1}{2}}.$$

B : In Lorentz invariance Violation Model

The dynamical attractor of the cosmological system has been occupied to make the late time manners of the model uncaring to the initial condition of the field and therefore alleviates the fine tuning problem. In quintessence models, the dynamical system has tracking attractor that makes the quintessence develops by tracking the equation of state of the background cosmological fluid so as to relieving the fine tuning problem.

Hence, in order to make viable Lorentz violation model, we need that the coupling function $\bar{\beta}$ and the potential function V must satisfy the condition

$$\frac{\bar{\beta}\bar{\beta}_{,\phi\phi}}{\bar{\beta}_{,\phi}^2} > 1/2$$

and

$$\frac{VV_{,\phi\phi}}{V_{,\phi}^2} + \frac{1}{2} \frac{\bar{\beta}_{,\phi}/\bar{\beta}}{V_{,\phi}/V} > 1$$

respectively.

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